Tuesday, February 7, 2017 11:30 PM

i>Clicker:

The moment of a couple is called a ______vector.

(A) Free

(B) Spinning

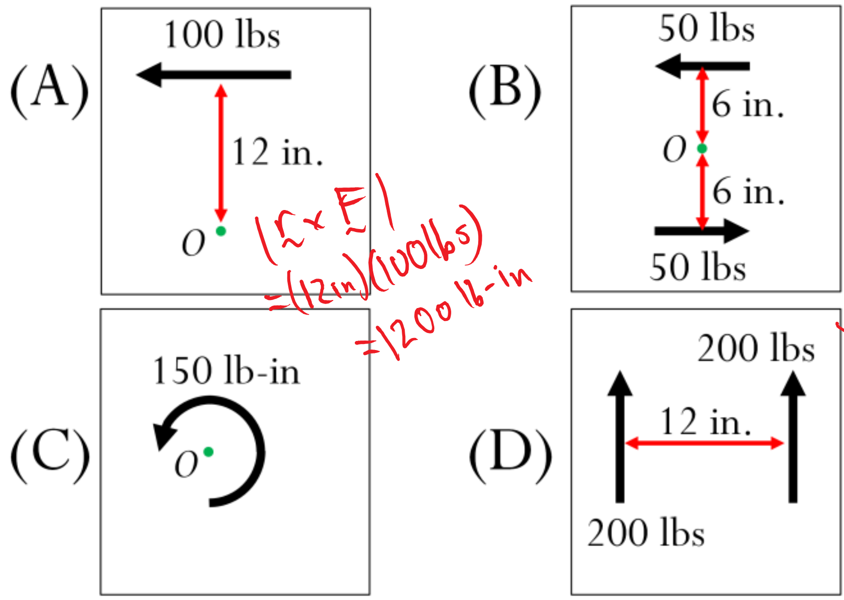
(C) Fixed

(D) Sliding

E) Other

it can be applied anywhere on a body with same external effect

i>Clicker: Which system has the largest moment?



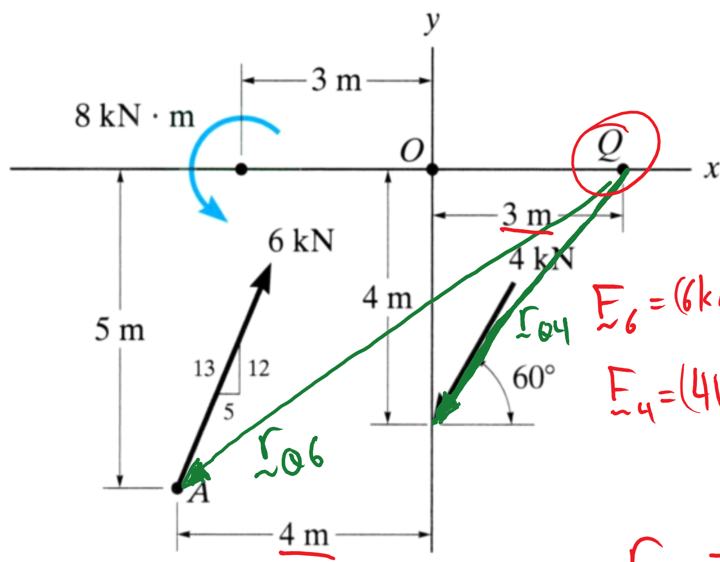
$|\underset{\sim}{r} \times \underset{\sim}{F}|$
$= (12\text{in})(100\text{lbs})$
$= 1200$ lb-in

$6 \cdot 50$ lb-in
$+ 6 \cdot 50$ lb-in
$= (300 + 300)$ lb-in
$= 600$ lb-in

$M_D = 0$

Problem

Replace the force and couple system by an equipollent force and couple moment at point Q.



$\underset{\sim}{F}_6 = (6kN)\left(\frac{5}{13}\hat{i} + \frac{12}{13}\hat{j}\right)$

$\underset{\sim}{F}_4 = (4kN)\left(\underbrace{-\cos 60°}_{-1/2}\hat{i} - \underbrace{\sin 60°}_{-\frac{\sqrt{3}}{2}}\hat{j}\right)$

$\underset{\sim}{r}_{Q6} = (-7\hat{i} - 5\hat{j})\,m$

$\underset{\sim}{r}_{Q4} = (-3\hat{i} - 4\hat{j})\,m$

Moments about Q:

$$M_{Q6} = \underset{\sim}{r}_{Q6} \times \underset{\sim}{F}_6 = (-7\hat{i} - 5\hat{j}) \times \left(\frac{30}{13}\hat{i} + \frac{72}{13}\hat{j}\right) kN\cdot m$$

$$= -27.23\,\hat{k}\ kN\cdot m$$

$$M_{Q4} = \underset{\sim}{r}_{Q4} \times \underset{\sim}{F}_4 = (-3\hat{i} - 4\hat{j}) \times (-2\hat{i} - 2\sqrt{3}\,\hat{j})\ kN\cdot m$$

$$= 2.392\,\hat{k}\ kN\cdot m$$

Find sum of all moments & couples

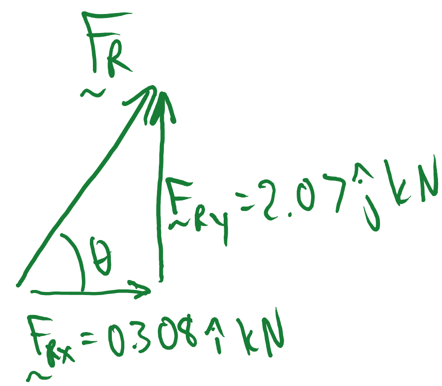
$$\sum M = \overbrace{(8\,kN\cdot m\,\hat{k})}^{\text{applied couple}} \overbrace{-(27.23\cdot kN\cdot m\cdot\hat{k}) + (2.392\cdot kN\cdot m\cdot\hat{k})}^{\sum \underset{\sim}{r}\times\underset{\sim}{F}}$$

$$= -16.84\cdot\hat{k}\cdot kN\cdot m$$

Resultant Force:

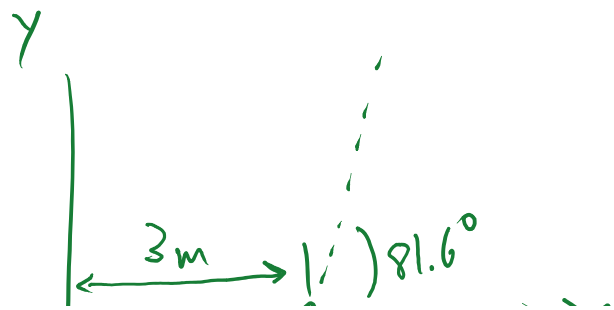
$$\underset{\sim}{F}_R = \underset{\sim}{F}_6 + \underset{\sim}{F}_4 = \left(\underline{0.308\,\hat{i} + 2.07\,\hat{j}}\right) kN$$

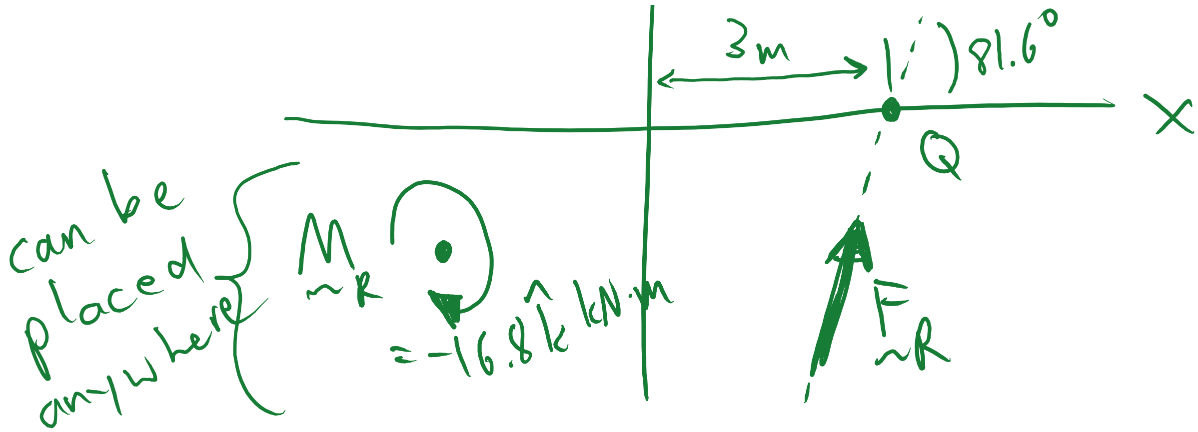
Find the line of action of $\underset{\sim}{F}_R$ through point Q



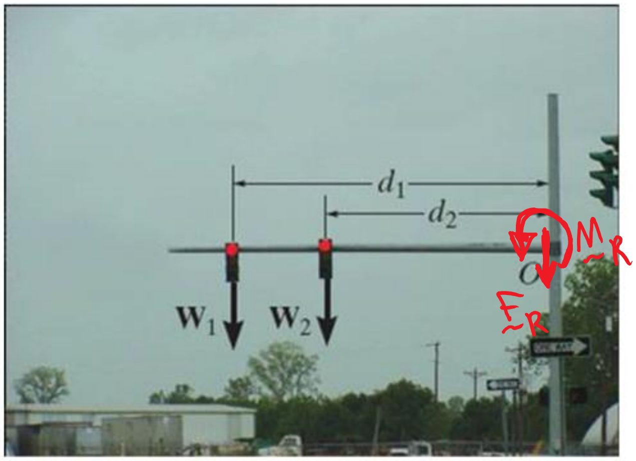
$$\theta = \tan^{-1}\left(\frac{2.07}{0.308}\right)$$

$$\theta = 81.6^\circ$$




can be placed anywhere
$\underset{\sim}{M}_R = -16.8\hat{k}$ kN·m
3m
81.6°
x
Q
$\underset{\sim}{F}_R$

What is the equivalent system?



$\hat{j}$ $\hat{i}$

Note that $\underset{\sim}{F}_R \perp \underset{\sim}{M}_R$

$$\underset{\sim}{F}_R = \sum \underset{\sim}{F} = -W_1 \hat{j} - W_2 \hat{j}$$

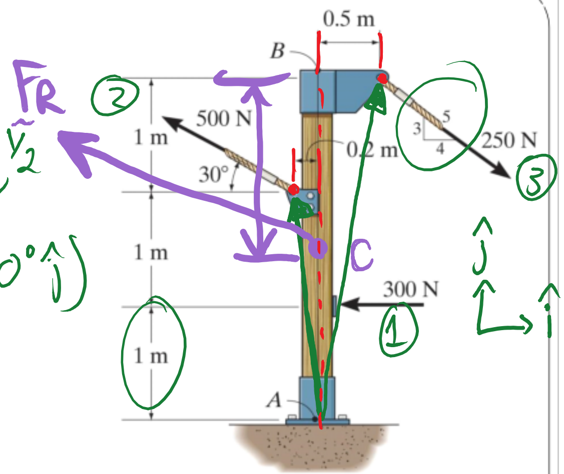
$$\underset{\sim}{M}_R = \sum \underset{\sim}{r} \times \underset{\sim}{F} + \underbrace{\sum \underset{\sim}{M}}_{\text{applied couples}}$$

$$= (d_1 \cdot W_1)\hat{k} + (d_2 \cdot W_2) \cdot \hat{k}$$

by right-hand rule, indicating C.C.W. tendency

$$\boxed{\begin{aligned} \underset{\sim}{F}_R &= (-W_1 - W_2)\hat{j} \\ \underset{\sim}{M}_R &= (d_1 W_1 + d_2 W_2)\hat{k} \end{aligned}}$$

Replace the force system acting on the post by a resultant force and resultant moment about point A.



$$\underset{\sim}{F}_R = \sum \underset{\sim}{F} = \overbrace{-300\,N\,\hat{i}}^{F_1}$$

$$+ (500N)(-\underbrace{\cos 30^\circ}_{\sqrt{3}/2}\,\hat{i} + \underbrace{\sin 30^\circ}_{1/2}\,\hat{j})$$

$$+ (250N)\left(\frac{4}{5}\,\hat{i} - \frac{3}{5}\,\hat{j}\right)$$

$$\underset{\sim}{F}_R = \left[\hat{i}\cdot(-300 - 250\sqrt{3} + 200) + \hat{j}\cdot(250 - 150)\right] N$$

$$\boxed{\underset{\sim}{F}_R \approx (-533\,\hat{i} + 100\,\hat{j})\,N}$$

$$\underset{\sim}{r}_1 = 1\,\hat{j}\ m$$

$$\underset{\sim}{r}_2 = (-0.2\,\hat{i} + 2\,\hat{j})\,m$$

$$\underset{\sim}{r}_3 = (0.5\,\hat{i} + 3\,\hat{j})\,m$$

$$\underset{\sim}{M}_R = \sum \underset{\sim}{M}_A = \sum \underset{\sim}{r} \times \underset{\sim}{F} = \underset{\sim}{r}_1 \times \underset{\sim}{F}_1 + \dots$$

$$= (1m\,\hat{j}) \times (-300N\,\hat{i}) + (-0.2m\,\hat{i} + 2m\,\hat{j}) \times (-250\sqrt{3}\,\hat{i} + 250\,\hat{j})\,N$$

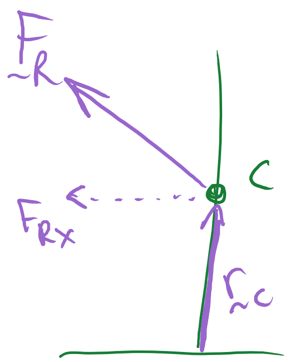
$$+ (0.5m\,\hat{i} + 3m\,\hat{j}) \times (200\,\hat{i} - 150\,\hat{j})\,N$$

$$\boxed{\underset{\sim}{M}_R \approx 441\,\hat{k}\ N\cdot m}$$

On what line does $\underset{\sim}{F}_R$ act to create a moment $\underset{\sim}{M}_R = 441\,\hat{k}\cdot N\cdot m$

(On the post)

Call it point C, which is located on the axis of the post $\Bigg\}$ Find C relative to point B



Find $\underset{\sim}{r}_c$ such that $\underset{\sim}{r}_c \times \underset{\sim}{F}_R = \underset{\sim}{M}_R$

$$\underset{\sim}{r}_c = (\cancel{x_c \hat{i}}^{\,0} + y_c \hat{j})$$

Scalar $\Rightarrow M_R = y_c \cdot F_{Rx}$

$$y_c = \frac{M_R}{F_{Rx}} = \frac{441 \, N\cdot m}{533 \, N} = 0.827 \, m$$ (measured up from A)

From pt. B: B is 3m above A

$$3m - 0.827m = \boxed{2.17m \text{ below point B}}$$

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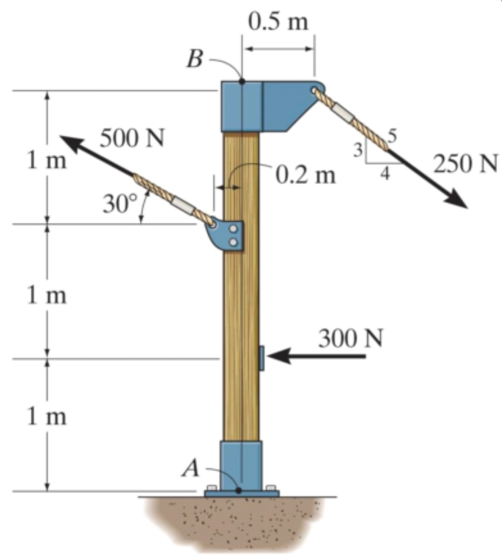
Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point B.

did not ask for a resultant moment.



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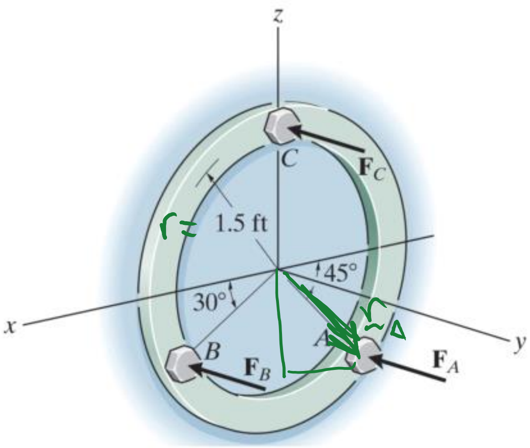

The three parallel bolting forces act on the circular plate, such that $F_A = F_B = F_C = F > 0$. Replace the force system by a resultant force $\boldsymbol{F}_R$ and a resultant moment $(\boldsymbol{M}_R)_O$ about the origin.

equal value

$$\underset{\sim}{F}_R = \sum \underset{\sim}{F} = \underset{\sim}{F}_A + \underset{\sim}{F}_B + \underset{\sim}{F}_C$$

$$\boxed{\underset{\sim}{F}_R = -3F \cdot \hat{j}}$$

Radius is $r = 1.5\text{ ft}$

$$\underset{\sim}{r}_A = -r \cdot \cos 45^\circ \hat{i} - r \cdot \sin 45^\circ \hat{k}$$

$$= -r \frac{\sqrt{2}}{2} (\hat{i} + \hat{k})$$

$$\underset{\sim}{r}_B = r \cdot \cos 30^\circ \hat{i} - r \cdot \sin 30^\circ \hat{k}$$

$$= \frac{r}{2} (\sqrt{3}\, \hat{i} - \hat{k})$$

$$\underset{\sim}{r}_C = r \cdot \hat{k}$$

$$\underset{\sim}{M}_R = \sum \underset{\sim}{r} \times \underset{\sim}{F} = -r \frac{\sqrt{2}}{2} (\hat{i} + \hat{k}) \times (-F \hat{j})$$

$$+ \frac{r}{2} (\sqrt{3}\, \hat{i} - \hat{k}) \times (-F \hat{j})$$

$$+ r \cdot \hat{k} \times (-F \hat{j})$$

$$= r \cdot F \cdot \left[\frac{\sqrt{2}}{2} (\hat{k} - \hat{i}) + \right.$$

$$= r \cdot F \cdot \left[\frac{\sqrt{2}}{2} \left(\hat{k} - \hat{i} \right) + \right.$$